EFFECTIVE DIFFUSION OF IMPURITY IN LAMINAR FLOW OF LIQUID

V. V. Dil'man

UDC 532.72.517.2

The effective diffusion of an impurity in a flow of non-Newtonian liquid is investigated.

The theory of the distribution of dynamically passive impurities in a laminar flow of Newtonian liquid was developed in [1]. In the present paper it is shown that the method of solution in [1] may be extended to the case of a non-Newtonian liquid. The velocity distribution over the tube cross section satisfies the law

$$V(r)=\frac{1+3n}{1+n}\bar{u}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]. \tag{1}$$

For n = 1, Eq. (1) reduces to the Poiseuille equation, describing the velocity profile in a laminar flow of Newtonian liquid.

Suppose that, in a certain cross section of the tube (x = 0), a batch of impurity is introduced into the liquid flow (the impurity is distributed uniformly over the whole cross section) and then its concentration is measured at a point lower down the flow. The impurity is assumed to be dynamically passive, i. e., to have no effect on the flow velocity, and the introduction of impurity does not disrupt the motion.

The distribution of impurity in the flow is determined by the balance between convective transfer along the tube and molecular diffusion. The differential equation for this process is

$$\frac{\partial C}{\partial t}+V(r)\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial x^2}\right). \tag{2}$$

Assume that the concentration changes more slowly along the x axis than over the tube radius, i. e., that

$$\frac{\partial^2 C}{\partial x^2}\ll\frac{\partial^2 C}{\partial r^2}. \tag{3}$$

In a frame of reference moving with mean flow velocity $\bar{u}$, the velocity of the liquid is given by the equation

$$V(r)=\frac{\bar{u}}{1+n}\left[2n-(1+3n)\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right], \tag{4}$$

and, taking Eq. (3) into account, the differential equation of the process takes the form

$$\frac{\partial C}{\partial t}+\frac{\bar{u}}{1+n}\left[2n-(1+3n)\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial C}{\partial r}\right). \tag{5}$$

In Eq. (5), the derivatives are taken in a moving frame of reference.

In the approximation $\partial C/\partial t$ = const and with the boundary condition

$$\left(\frac{\partial C}{\partial r}\right)_{r=R}=0 \tag{6}$$

the steady solution of Eq. (5) is

$$C=\bar{C}+\frac{n}{2(1+n)}\cdot\frac{\bar{u}R^2}{D}\cdot\frac{\partial\bar{C}}{\partial x}\left\{\left(\frac{r}{R}\right)^2-\left[\frac{1}{2}-\frac{4n^2}{(1+3n)(1+5n)}\right]-\frac{2n}{1+3n}\left(\frac{r}{R}\right)^{\frac{1+3n}{n}}\right\}. \tag{7}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 2, pp. 342-345, February, 1977. Original article submitted February 23, 1976.

The flux density of impurity passing through the tube cross section is

$$j = 2\int_0^1 C(\xi)V(\xi)\xi d\xi = -D_{\text{ef}}\frac{\partial \bar{C}}{\partial x}, \quad (8)$$

where

$$D_{\text{ef}} = \frac{n^2}{2(1+3n)(1+5n)}\,\frac{\bar{u}^2R^2}{D}. \quad (9)$$

In the next approximation, assuming that $\partial\bar{C}/\partial x = \text{const}$,

$$\frac{\partial C}{\partial t} = -\frac{\partial j}{\partial x} = D_{\text{ef}}\frac{\partial^2 \bar{C}}{\partial x^2}. \quad (10)$$

The result obtained shows that, within the framework of the assumptions made, the distribution of the impurity along the x axis is described by the diffusion equation.

It remains to consider the restriction of the problem due to the assumption of slow change in impurity concentration along the longitudinal coordinate [see Eq. (3)].

According to Eq. (7), the derivative of the concentration with respect to x is

$$\frac{\partial C}{\partial x} = \frac{\partial \bar{C}}{\partial x} + \frac{n}{2(1+n)}\cdot\frac{\bar{u}R^2}{D}\cdot\frac{\partial^2\bar{C}}{\partial x^2}\left\{\left(\frac{r}{R}\right)^2 - \left[\frac{1}{2} - \frac{4n^2}{(1+3n)(1+5n)}\right] - \frac{2n}{1+3n}\left(\frac{r}{R}\right)^{\frac{1+3n}{n}}\right\}. \quad (11)$$

It follows from Eq. (11) that the condition $\partial C/\partial x = \text{const}$ leads to an inequality which, on the flow axis $(r/R = 0)$, can be written in the form

$$\frac{\partial\bar{C}}{\partial x} \gg \frac{n(1+8n+7n^2)}{4(1+n)(1+3n)(1+5n)}\cdot\frac{\bar{u}R^2}{D}\cdot\frac{\partial^2\bar{C}}{\partial x^2}.$$

Let L be the length along the flow axis in which marked change in the impurity concentration occurs. Then the above inequality takes the form

$$\frac{LD}{R^2\bar{u}} \gg \frac{n(1+8n+7n^2)}{4(1+n)(1+3n)(1+5n)}. \quad (12)$$

Further restriction on the problem results from the assumption $D \ll D_{\text{ef}}$.

From Eq. (9)

$$\frac{R\bar{u}}{D} \gg \sqrt{\frac{2(1+3n)(1+5n)}{n^2}}. \quad (13)$$

Since

$$\frac{n(1+8n+7n^2)}{4(1+n)(1+3n)(1+5n)} < 1,$$

Eqs. (12) and (13) give the inequality

$$\frac{L}{R} \gg \frac{R\bar{u}}{D} \gg \sqrt{\frac{2(1+3n)(1+5n)}{n^2}}, \quad (14)$$

which allows the limits of applicability of Eq. (10) to be established.

If the rheological properties of the flow are known, i. e., if the value of n in Eq. (1) is known, then determining the value of D_{ef} by the method of [2] permits the calculation of the flow velocity (if D is known) or of D (if the flow velocity is known).

In conclusion, note that when n = 1 the present solution reduces to that in [1].

NOTATION

V(r), local liquid velocity; $\bar{u}$, liquid velocity averaged over tube cross section; n, flow coefficient; x, longitudinal coordinate; r, radius at a certain point; R, tube radius; $\xi = r/R$, dimensionless coordinate; L, length along flow axis; t, time; C, impurity concentration; D, coefficient of molecular diffusion; D_{ef}, effective-diffusion coefficient.

LITERATURE CITED

1. G. Taylor, Proc. Roy. Soc., A219, No. 1137, 186 (1953).
2. V. V. Dil'man, M. B. Aizenbud, and É. Z. Shul'ts, Khim. Prom., No. 2 (1966).

MATHEMATICAL MODELING OF PARTICLE GROWTH

L. A. Bakhtin, N. A. Shakhova,
A. I. Pronin, N. A. Kudryavtsev,
Ya. M. Shul'man, and L. A. Kamneva

UDC 532.529.5

Continuity equations for particle distributions by size and residence times are considered in processes associated with particle growth. The relation between these equations and the particle-balance equation in phase space is shown.

Particle-balance equations (continuity equations for particle distributions) occupy an important place in the study of processes associated with particle growth [1-11]. The most general approach to the formulation of such equations was outlined in [12, 13], which proposed the description of a heterogeneous process associated with any transformation of particles of a disperse phase, such as the motion of a point reflecting the state of the particle in a multidimensional phase space (a phase space is taken to be a system of spatial coordinates and coordinates characterizing the internal state of the particle). The particle-balance equation in this case is

$$\frac{\partial \rho_1}{\partial t} + \operatorname{div}(\vec{\omega}\rho_1) + \sum_{k=1}^{m} \frac{\partial}{\partial \xi_k}\left(\rho_1 \frac{d\xi_k}{dt}\right) = \psi_1, \tag{1}$$

where $\rho_1 = \rho_1(x, y, x, \xi_1, \ldots, \xi_m)$ is the density of the particle distribution.

For a nonideal system, terms characterizing particle mixing must be introduced in Eq. (1). For example, if particle mixing proceeds by the diffusion law, the appropriate equation is

$$\frac{\partial \rho_1}{\partial t} + \operatorname{div}(\vec{\omega}\rho_1 - D \operatorname{grad} \rho_1) - \sum_{k=1}^{m} \frac{\partial}{\partial \xi_k}\left(\rho_1 \frac{d\xi_k}{dt}\right) = \psi_1. \tag{2}$$

In processes of particle growth, the particle size serves as the internal coordinate. If growth is accompanied by other processes (drying, chemical change, etc.), coordinates characteristic for these processes (moisture content, degree of transformation, etc.) must be introduced into the equation. As a rule, however, all these parameters may be represented as different functions of a single variable — τ, the residence time of the particle in the apparatus. Thus, Eq. (2) may be written in the form

$$\frac{\partial \rho_2}{\partial t} + \operatorname{div}(\vec{\omega}\rho_2 - D \operatorname{grad} \rho_2) + \frac{\partial \rho_2}{\partial \tau} = \psi_2, \tag{3}$$

where $\rho_2 = \rho_2(x, y, z, \tau)$.

It can easily be shown that several known solutions of the balance equations in processes of particle growth [1-11] are partial cases of the solution of Eqs. (2) and (3) for conditions of ideal mixing and ideal substitution. In the present work, an attempt is made to solve the continuity equation of the particle distribution in the diffusion model for nonideal conditions.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 2, pp. 346-349, February, 1977. Original article submitted February 5, 1976.